Differential Amplification

One of the desired characteristics of the operational amplifier was having differential inputs -- two inputs, one inverting and the other non-inverting. So far, we've seen that this allows us to create both inverting and non-inverting amplifiers. But that's only scratching the surface of what can be done with differential inputs on an amplifier.

If equal but opposite signals are applied to the two inputs of a differential amplifier, the output signal is amplified twice as much as it would have been for a single-input amplifier. The gain of a differential amplifier is

$$A_{v(diff)} = \frac{V_{out}}{(V_{in+} - V_{in-})}$$

Rearranged in terms of the output voltage,

$$V_{out} = A_{v(diff)} \left(V_{in+} - V_{in-}\right)$$

To explain the difference in output signal size for a single-input amplifier *vs*. a differential input amplifier, consider the following example:

If the input signal to an amplifier with a gain of 10 was +1.0 V:

- applying the signal to the non-inverting input while the inverting input was at ground would result in V_{out} = 10*(1.0 - 0) = 10 V
- applying the signal to the inverting input with the non-inverting input at ground would result in V_{out} = 10*(0 - 1.0) = -10 V
- applying the signal to the non-inverting input and the inverse of the signal, -1.0 V, to the inverting input would result in V_{out} = 10*(1.0 - -1.0) = 20 V

We often refer to the difference of the two inputs as the differential input voltage, V_{diff}.

$$V_{diff} = V_{in+} - V_{in-}$$

Common Mode Rejection

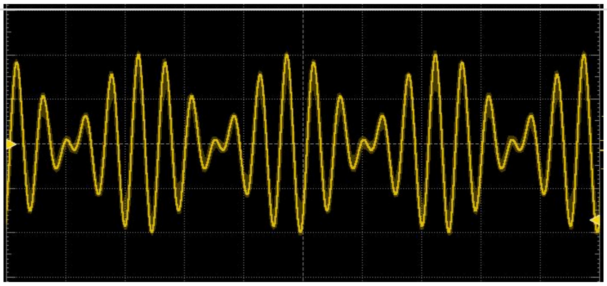
Just as significant as Differential Amplification is something called Common Mode Rejection. This refers to what happens to an identical signal that is applied to both inputs of a differential amplifier. Again, consider our previous example with a gain of 10 and an input signal of 1 V. If the same signal is applied to both inputs, the result is V_{out} = 10*(1.0 - 1.0) = 0 V. In other words, any signal that is "common" to both inputs will be removed from the output! Thus, ***common mode signals*** will be rejected by the differential amplifier. This may seem like a waste, but in reality it is one of the amazing features of the operational amplifier, when used as a differential amplifier. Consider the following two examples:

- In a concert facility, microphone and instrument signal cables are routed from the stage to a soundboard at the back of the room, crossing over AC power cords and under flourescent lights, all of which are operating at 60 Hz, and injecting noise into the signal cables. If the cables are single-ended (signal referenced to ground), the noise on the signal-carrying line will be amplified along with the desired signal, resulting in an annoying hum; however, if the cable is differential, the desired signal will be doubled while the unwanted common mode signal will be eliminated!
- Certain industrial sensors, such as Thermistors and Resistance Temperature Detectors (RTDs) must typically be wired so that the resulting signal has a large DC component and a small useful signal. If an equal DC signal is connected to the other terminal of a differential amplifier, the DC component will be eliminated, leaving only the desired small signal.

The common mode part of a combined signal can be determined as follows:

$V_{in(CM)} = \frac{V_{in+} + V_{in-}}{2}$ in other words, the average of the two inputs.

It seems a little silly to state these two formulas to describe differential and common mode signals. However, when it comes to the kinds of signals that are presented to an amplifier, they are rarely simple voltages or simple true AC signals -- they will be combinations of frequencies and DC components, all of which must be treated separately. For example, a 1.0 V_p 100 Hz signal and a 1.0 V_p 120 Hz signal can't be combined and called a 2.0 V_p signal at either of the frequencies or any combination of the frequencies. Here's what the result looks like:



In places, the amplitude is 2.0 V_p, and in places the amplitude is 0.0 V_p. The frequency is neither 100 Hz not 120 Hz, and there's a repetitive pattern called a "beat frequency" that happens every 50 ms for a "buzzing" sensation at 20 Hz. The main point of this is that we have to deal with each of the components separately in our analysis of a complex signal. Let's do an example.

On a particular differential pair of wires, the following components have been identified. Notice, in this case, that we're allowed to use a negative V_p because it indicates a 180° phase difference between two related signals.

V_+ = 1.2 V_p @ 250 Hz + 0.8 V_p @ 60 Hz + 0.50 V_{DC}

V_- = -1.2 V_p @ 250 Hz + 0.8 V_p @ 60 Hz - 0.20 V_{DC}

1. V_{diff} = 2.4 V_p @ 250 Hz + 0 V_p @ 60 Hz + 0.7 V_{DC}
2. V_{CM}= 0 V_p @ 250 Hz + 0.8 V_p @ 60 Hz + 0.15 V_{DC}

Notice that equal amplitude components are easy to deal with, but unequal components don't disappear in either the differential term or the common mode term. This leads somewhat indirectly to the next topic.

Common Mode Rejection and Common Mode Rejection Ratio

No amplifier is perfect, and no pair of signal wires will be truly identical; so some portion of the undesirable common mode signal is likely to be transmitted through the amplifier. The common mode gain, therefore, will be

$$A_{v(CM)} = \frac{V_{o(CM)}}{V_{in(CM)}}$$

Hopefully, the common mode gain will be really small. In other words, the common mode component should be significantly attenuated. If the differential gain is really big, then the result will be a pretty clean signal. We quantify this using the ***Common Mode Rejection Ratio***.

$$CMRR = \frac{A_{v(diff)}}{A_{v(CM)}}$$

Since the differential gain should be so much bigger than the common mode gain, we usually state this in decibels as ***Common Mode Rejection***.

$$CMR = 20\ log CMRR$$

3. For a particular differential amplifier, the following values were measured:

- V_+ = 22.5 mV_p @ 3.0 kHz + 3.0 V_{DC}
- V_- = -22.5 mV_p @ 3.0 kHz + 3.0 V_{DC}
- V_{out} = 6.0 V_p @ 3.0 kHz + 20 mV_{DC}

What is the Differential Gain? 133

What is the Common Mode Gain? 0.0067+5E-4

What is the Common Mode Rejection Ratio? 20000

What is the Common Mode Rejection? 86 dB

Notice, in this example, how a tiny differential signal and a large common mode signal, once passed through this amplifier, produce a large version of the differential signal and an almost non-existent common mode component. That's what differential amplification and common mode rejection are all about.