The frequency limitations of the op amp turn all op amp circuits into low pass filters. However, when it comes to controlling frequency response, we don't usually want to rely on the amplifier's internal characteristics -- we want to build our own filters with predictable characteristics.

When it comes to filters, we use the term "Passive Filter" to refer to a filter made using just capacitors, resistors, and inductors. We use the term "Active Filter" to refer to any filter that uses op amps or transistors as well. We'll stick to op amp circuits in this course.

The simplest Active Filters add only one capacitor to the inverting amplifier configuration we've been using.

Single-Pole Low-Pass Filter

Add a 15 nF in parallel with R2 in your Multisim circuit. Set your function generator to produce a sine wave 500 $mV_{p\text{-}p}$ at a frequency of 100 Hz.

1. Measure the output voltage, and from this calculate the "target voltage" for the cutoff frequency. [3.54] $V_{p\text{-}p}$
2. Now, increase the frequency until you hit the target voltage. Record the cutoff frequency: [1.06] kHz.
3. Given that cutoff frequencies are always based on the formula $f_c=1/(2\pi RC)$, which of the two resistors works with the capacitor to produce this cutoff frequency?

- ○ Ri
- ◉ Rf

Some of you may be wondering why we put the capacitor in parallel with Rf instead of putting it from R1 to ground, the way you did when you made passive filters last semester. That won't work for an inverting op amp configuration, because, due to the virtual short, the inverting pin is at zero volts, so a capacitor connected here would have zero volts at both ends and would have no effect on the signal! So, for a simple active low-pass filter, the cutoff frequency can be predicted using

$$f_c = \frac{1}{2\ \pi R_f C}$$

Single Pole High-Pass Filter

Move the capacitor so that it is in series with Ri instead. It doesn't matter which side, but we usually put it next to the inverting pin. Notice that having a capacitor in series with the input means that DC would be blocked. That's a quick way to determine that this is a High Pass Filter, since DC is the lowest frequency possible, so blocking DC means blocking low frequencies and passing high frequencies.

Set the function generator initially to produce a sine wave 500 $mV_{p\text{-}p}$ initially at 30 kHz.

4. Again, measure the output voltage and from this calculate the "target voltage" for the cutoff frequency [3.54] $V_{p\text{-}p}$
5. This time, DECREASE the frequency until you hit the target voltage, since the pass band for a HPF is ABOVE the cutoff frequency. Record the cutoff frequency. [10.6] kHz
6. Again, since cutoff frequencies are always based on a resistor and a capacitor, which of the two resistors works with the capacitor to product this cutoff frequency?

- ◉ Ri
- ○ Rf

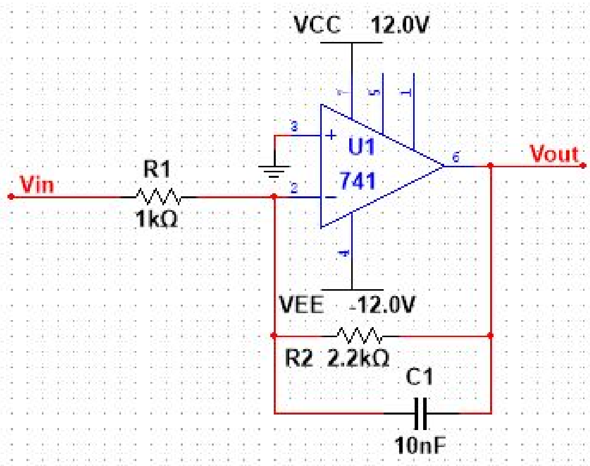
So, for a simple active HPF, the cutoff frequency can be predicted using

$$f_c = \frac{1}{2\ \pi R_i C}$$

For all filters, if the signal is within the pass-band, the capacitor can be ignored so that the rest of the circuit can be treated simply as an amplifier. So, for both of these circuits as you have seen, the gain in the pass band is just

$$A_v = -\frac{R_f}{R_i}$$

Worked Examples



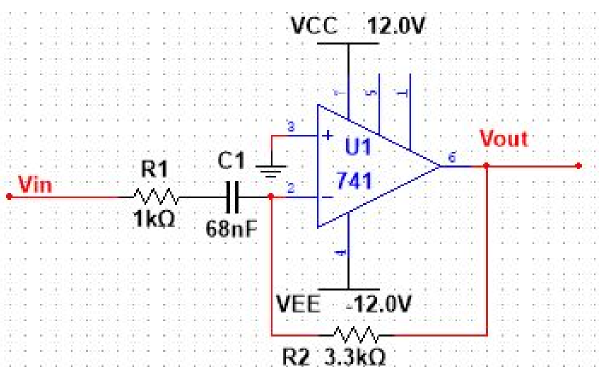
Quick analysis: this is an inverting amplifier, and the capacitor in parallel with the feedback resistor makes it a Low-Pass Filter (LPF).

Its passband gain would be -2.2kΩ/1.0kΩ = -2.2

The predicted cutoff frequency would be 1/(2*π*R_f*C) = 7.23 kHz.

If the input signal was a sinewave with an amplitude of 250 mV_p:

- at 700 Hz (about a decade inside the passband, which is from 0 Hz to 7.23 kHz) the output amplitude would be -2.2*250 mV_p = 550 mV_p, inverted
- at 7.23 kHz (the cut-off frequency) the output amplitude would be 550 mV_p/ sqrt(2) = 389 mV_p
- at 72.3 kHz (one decade past the cut-off frequency) the output amplitude would be 550 mV_p/10 = 55 mV_p
- at 14.26 kHz (one octave past the cut-off frequency) the output amplitude would be 550 mV_p/2 = 275 mV_p
- to determine the cut-off frequency empirically:
  - set the frequency to about 700 Hz (a decade below the expected cut-off frequency)
  - measure the output voltage, $V_{out(pass)}$
  - determine the target voltage: V_{fc} = $V_{out(pass)}$/sqrt(2)
  - INCREASE the frequency until the amplitude drops to V_{fc}
  - record the frequency as f_c



Quick analysis: this is an inverting amplifier, and the capacitor in series with the input resistor makes it a High Pass Filter (HPF).

Its passband gain would be -3.3kΩ/1kΩ = -3.3

The predicted cut-off frequency would be 1/(2*π*R_i*C) = 2.34 kHz

If the input signal was a sinewave with an amplitude of 1.0 V_p:

- at 23 kHz (about a decade within the passband, which is 2.34 kHz and up) the output amplitude would be -3.3*1.0 V_p = 3.3 V_p, inverted
- at 2.34 kHz (the cut-off frequency) the amplitude would be 3.3 V_p/sqrt(2) = 2.33 V_p
- at 234 Hz (one decade before the cut-off frequency) the amplitude would be 330 mV_p (one tenth)

- at 1.17 kHz (one octave before the cut-off frequency) the amplitude would be 1.65 V_p (one half)
- to determine the cut-off frequency empirically:
    - set the frequency to 23 kHz (about a decade within the pass-band)
    - measure $V_{out(pass)}$
    - determine the target voltage from $V_{fc} = V_{out(pass)}$/sqrt(2)
    - DECREASE the frequency until the voltage drops to the target voltage, V_{fc}
    - record the frequency as f_c

Multi-Pole Butterworth Sallen-Key Filters

The problem with the circuits we've designed is that they only have a 20 dB/decade rolloff. This means that they don't very effectively remove frequency components close to the cutoff frequency. To improve on this, we need to introduce more capacitors. At a given frequency, having additional R-C pairs increases the rolloff by

20 dB/decade per capacitor

also known as

20 dB/decade per pole.

The term "pole" comes from an infinitely high spike that shows up in the 3-D impedance graph for a capacitor in a filter.

The problem with just adding more R-C pairs is that they cause instability in the circuit. Many different active filter designs have been made that overcome this problem. We'll just teach you one well-known one that produces very dependable results -- one designed by two people whose last names were Sallen and Key. The Sallen-Key Filter is also known as the VCVS Filter (Voltage-Controlled Voltage Source).

Having op amps in a circuit means that, unless things are done just right, the circuit could go into oscillation. What we're looking for in this course is what's called a "maximally flat" response, or a Butterworth response. With this arrangement, the output voltage stays constant until as close as possible to the cutoff frequency, then rounds down in the shape expected for a single capacitor filter before dropping at the predicted rolloff rate. These filters are called "Critically Damped" as opposed to "Underdamped" filters, which oscillate a bit near the cutoff frequency, or "Overdamped" filters which start to round down well before the cutoff frequency.

For a Butterworth Sallen-Key Filter, the required gains are defined by a group of S-Domain polynomials called the "Normalized Butterworth Polynomials", shown below.

n	**Normalized Butterworth Polynomial**
1	$s+1$
2	$s^2+1.414s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.765s+1)(s^2+1.848s+1)$
5	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$
6	$(s^2+0.518s+1)(s^2+1.414s+1)(s^2+1.932s+1)$

In this table, the monomials (s+1) all represent single-pole filter components. The binomials all represent two-pole filter components, which we will look at soon. The single pole components can have any gain we want them to have. However, the binomial components must have a voltage gain, A_v, that is equal to 3 minus the middle term coefficient. So, for example, the gains required for the three parts of a 5-pole filter would be

Stage 1: X (any gain)

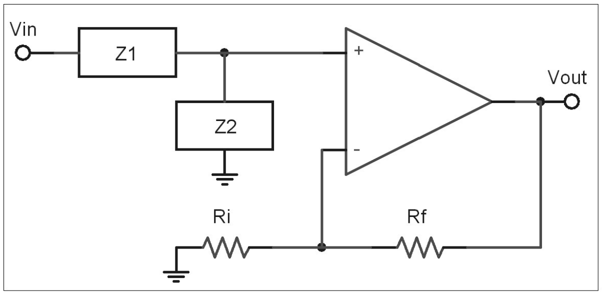
Stage 2: 2.382 (that's from 3 - 0.618)

Stage 3: 1.382 (that's from 3 - 1.618)

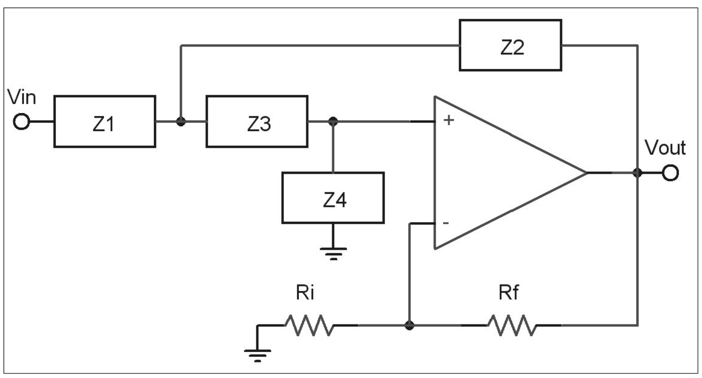
The last column in this table (which also appears in this course's formula sheet) shows the resulting gains for multi-pole filters up to six poles.

n	Normalized Butterworth Polynomial	Stage Gains
1	$s+1$	(x)
2	$s^2+1.414s+1$	(1.586)
3	$(s+1)(s^2+s+1)$	(x)(2.000)
4	$(s^2+0.765s+1)(s^2+1.848s+1)$	(2.235)(1.152)
5	$(s+1)(s^2+0.618s+1)(s^2+1.618s+1)$	(x)(2.382)(1.382)
6	$(s^2+0.518s+1)(s^2+1.414s+1)(s^2+1.932s+1)$	(2.482)(1.586)(1.068)

Now, for the actual circuits that embody these polynomials.



-- Single-Pole Component



-- Two-Pole Component

In each case, the "Z" elements are either resistors or capacitors. Z1 and Z3 have to be the same type of component, and Z2 and Z4 have to be the other type of component. If the odd-numbered ones are resistors, the result is a LPF. If the odd-numbered ones are capacitors, the result is a HPF.

If all the capacitors and all the resistors in the filter-related feedback (the Zs) are the same, the cutoff frequency is the usual:

$$f_C = \frac{1}{2\ \pi RC}$$

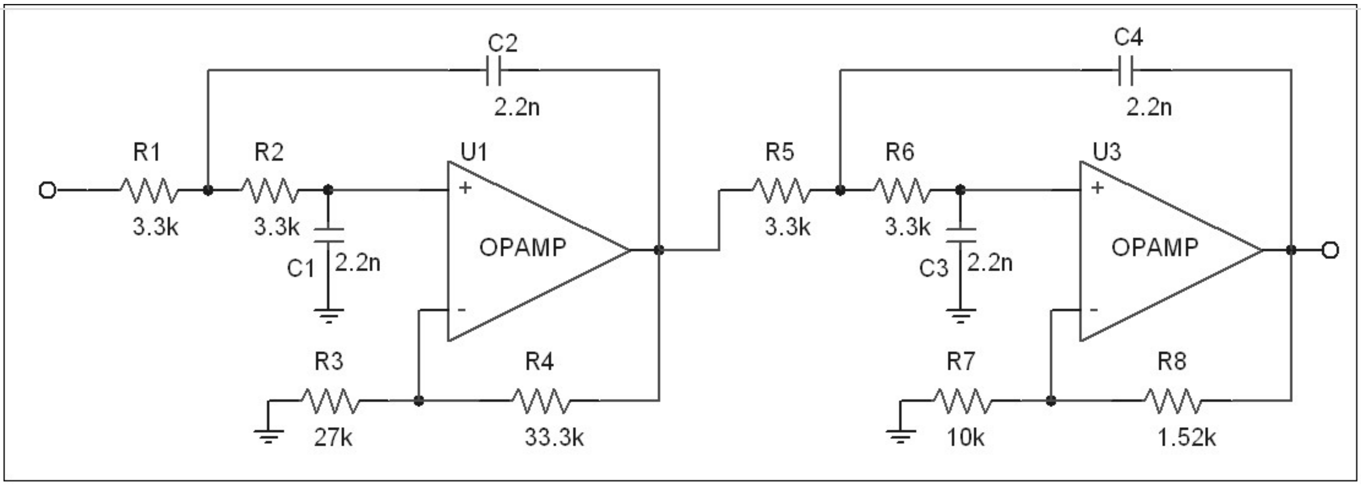
Lucky for you, we won't ever use combinations of different resistors and capacitors in the filter-related feedback.

In the negative feedback component, the gain is what you would expect for a non-inverting amplifier:

$$A_v = \frac{R_f}{R_i} + 1$$

The big limitation, though, is that these gains must match the ones in the table in order to produce a Butterworth, or maximally flat, characteristic.

Here's an example:



7. What type of filter is this?

- (selected) Low Pass
- High Pass
- Band Pass
- Band Reject

8. How many poles does it have (also known as the "order" of the filter)?

4

9. What is the expected rolloff past the cutoff frequency?

80 dB/decade

10. What is the cutoff frequency? 21.9 kHz

11. What is the pass band gain for the first stage? 2.233

12. What should it be, from the table? 2.235

13. What is the pass band gain for the second stage? 1.152

14. What should it be? 1.152

These gains are pretty close to optimal, so this filter should be maximally-flat.

15. What is the overall passband gain of the circuit? 2.572

This last question points out one of the limitations of even-ordered filters -- their overall gain can't be played with. However, the odd-ordered filters have a single-pole stage, so the gain can be set to what you wish it to be.

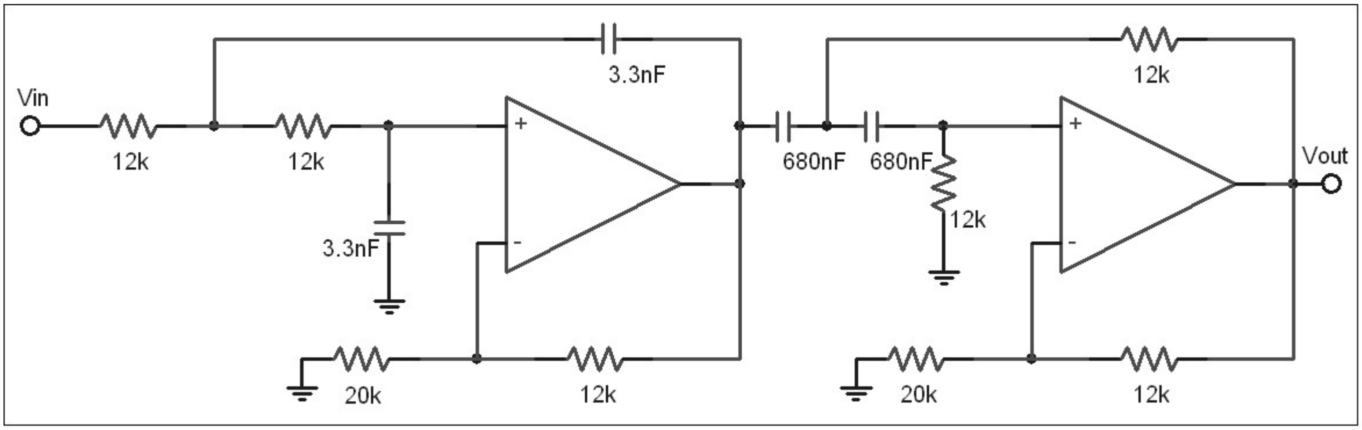
Band Pass and Band Reject filters

A Band Pass Filter is made by putting a HPF in SERIES with a LPF, and setting the cutoff frequency of the LPF to be HIGHER than the cutoff frequency of the HPF. That way, there will be a pass band of frequencies between the two cutoffs.

A Band Reject Filter is made by adding the outputs of a HPF filter and a LPF filter using a summing amplifier. That way, the two pieces of the spectrum that you want are joined together in the final output signal.

It's important to realize that you can't just add up the capacitors to determine how many poles a BPF or BRF have, since the two components of the filter are active at different frequencies.

Here's an example.



16. If the cutoff frequencies are chosen properly, what kind of filter is this?

- ( ) LPF
- ( ) HPF
- (•) BPF
- ( ) BRF

17. What is the cutoff frequency for the first stage? 4.02 kHz

18. What is the cutoff frequency for the second stage? 19.5 Hz

- (•) True
- ( ) False

19. The cutoff frequencies are chosen to allow a range of pass band frequencies.

20. How many poles are there in the LPF component? 2

21. How many poles are there in the HPF component? 2

22. What is the expected rolloff for the filters in this circuit? 40 dB/decade

23. Notice that the two components have the same gain-setting resistors. What is the gain of each section?

1.6

24. What should the gain be, from the table? 1.586

25. The gains should be close enough to be maximally-flat, or Butterworth.

26. What is the overall gain of the circuit? 2.56

That's a lot of information! Hard as it may be to believe, you've only scratched the surface of the topic of frequency response. These are the most important things to remember:

- Filters remove, or at least attenuate, signals outside of their pass bands
- Low Pass Filters remove high frequencies
- High Pass Filters remove low frequencies
- Band Pass Filters remove low and high frequencies, passing a "mid" band
- Band Reject Filters keep low and high frequencies, rejecting a "mid" band
- The cutoff frequency is found empirically by changing the frequency until the output signal drops to half power, or V/sqrt(2)
- The cutoff frequency can be predicted using $f_c=1/(2\pi RC)$
- In the rejected region, the amplitude drops off at 20 dB/decade per pole
- Sallen-Key filter stage gains must match the table of polynomials for maximally-flat or Butterworth response